

# Spontaneous magnetization of a vacuum in the hot Universe and intergalactic magnetic fields

V. Skalozub<sup>a</sup>

<sup>a</sup> *Dnipropetrovsk National University, Ukraine*

# ABSTRACT

It is discussed that long range coherent magnetic fields in the Universe were spontaneously generated at high temperature due to vacuum polarization of non-Abelian gauge fields. The fields created at the reheating have resulted in the present intergalactic magnetic field  $B \sim 10^{-15}G$ . The zero value of the screening mass for fields of this type was discovered recently. A procedure to estimate the field strengths at different temperatures is developed and the lower bound on the magnetic field strength  $B \sim 10^{14}G$ , at the electroweak phase transition temperature, is derived for the standard model. The magnetic field scale is estimated.

# OUTLINE

- INTERGALACTIC MAGNETIC FIELDS
- MECHANISMS FOR GENERATION of MAGNETIC FIELDS IN HOT UNIVERSE
- SPONTANEOUS VACUUM MAGNETIZATION at HIGH TEMPERATURE
- MAGNETIC FIELD CHARACTERISTICS
- QUALITATIVE CONSIDERATION of PHENOMENA
- EFFECTIVE POTENTIAL at HIGH TEMPERATURE
- MAGNETIC FIELD at  $T_{ew}$
- MAGNETIC FIELD SCALE
- CONCLUSION

# INTERGALACTIC MAGNETIC FIELDS

Magnetic fields  $B \sim \mu G$  presence everywhere – in galaxies, clusters of galaxies

Determination of intergalactic magnetic fields  $B_0 \sim 10^{-15} G$ :

[S. Ando, A. Kusenko, *Astrophys. J. Lett.* **722** (2010) L 39][arXiv:1005.1924] looked at the source morphology (halo,  $\gamma$  cascades:  $\gamma \rightarrow e^+e^- \rightarrow \gamma^*, \gamma^*, \dots$ );

[S. Ando, A. Kusenko, [arXiv:1012.5313]] looked at blazer spectra.

Complementary and independent methods.

This value was estimated either as lower or upper limit. So, it is actual value at 3.5CL accuracy.

[A. Neronov, E. Vovk. *Science* **328** (2010) 73.]  $B_0 \sim 10^{-16} G$ .

To be amplified by dynamo action of large-scale convective motions the fields must be coherent on the scale  $1\text{mPc}$ .

Zeeman splitting and/or Faraday rotation are operating if  $B \geq 10^{-9}G$ .

## ASTROPHYSICAL CONSTRAINTS

- Big Bang Nucleosynthesis (BBN) limit  $B \leq 10^{11}G$   
or  $B \leq 7 \cdot 10^{-7}G$  at galaxy formation;
- Cosmic microwave background (CMB) limit  $B \leq 10^{-9}G$ .

# MECHANISMS for GENERATION of B

Popular mechanisms for generation of seed magnetic fields at high temperature in the early universe:

- metric perturbations
- strong first order EW phase transition [Hogan (1980)]
- stochastic electric currents
- paramagnetic resonances in scalar (or axion) - electromagnetic field system
- Born-Infeld electrodynamics, HE effective Lagrangian
- inflation
- cosmic strings
- trace anomaly
- extradimensions
- gravitational couplings of gauge field potentials

In all these considerations it is **ASSUMED**

*magnetic flux is conserved* and therefore the dependence of  $B \sim T^2$  takes place at cooling of the universe.

## **OUR MAIN IDEA:**

seed (primordial) magnetic field is *spontaneously generated* at high temperature due to

*vacuum polarization and asymptotic freedom* of non-Abelian gauge fields.

These magnetic fields are temperature dependent

[Starinets, Vshivtsev, Zhukovsky(1994)],

[Skalozub(1996)], [Bordag, Skalozub (2000)],

[Demchik, Skalozub (2008)] (**in lattice simulations**):

$$B(T) \sim \frac{g^3 T^2}{\log \frac{T}{\tau}}. \quad (1)$$

So, **there is no magnetic flux conservation at high temperature!**

At zero temperature, [Savvidy (1978)]. The magnetized vacuum state is unstable because of the mode  $p_0^2 = p_{\parallel}^2 - gB$  in the gluon spectrum,

$$p_0^2 = p_{\parallel}^2 + (2n + 1)gB, \quad n = -1, 0, 1, \dots, \quad (2)$$

that results in a condensate. Because of instability, the Abelian constant magnetic field  $B = \text{const}$  is completely screened.

At  $T \neq 0$  the spectrum stabilization happens due to either a gluon magnetic mass [Bordag, Skalozub (2000)] or a so-called  $A_0$ -condensate which is proportional to the Polyakov loop [Starinets, Vshivtsev, Zhukovskii (1996)]. That is implemented in a stable magnetized vacuum.



As it was shown [Skalozub, Strelchenko (1999)], this mass is positive and stabilizes the spectrum at high temperature:

$$m_{eff.}^2 = m_{magn.}^2 - gB > 0, \quad m_{magn.}^2 \sim g^2(gB)^{1/2}T. \quad (3)$$

In the same way the  $A_0$ -condensate acts in the high temperature phase. Thus, in the deconfinement phase a spontaneously generated Abelian chromomagnetic field could happen.

The Abelian chromomagnetic field directed in the third direction in a coordinate and internal space can be described by the potential

$$A_\mu^a = \delta^{a3}(0, 0, Bx^1, 0), \quad B = const. \quad (4)$$

It is a solution of field equations without a source term.

So, **it can be spontaneously generated.**

# SPONTANEOUS VACUUM MAGNETIZATION at HIGH TEMPERATURE

On a lattice, the main continuous object is a magnetic flux. We relate the free energy density of the flux to the effective action [Demchik, Skalozub (2008)],

$$F(\varphi) = \bar{S}(\varphi) - \bar{S}(0), \quad (5)$$

where  $\bar{S}(\varphi)$  and  $\bar{S}(0)$  are the effective lattice actions with and without chromomagnetic field,  $\varphi$  is the field flux.

**The spontaneous creation of the field follows if free energy has a global minimum at non-zero flux,  $\varphi_{min} \neq 0$ .**

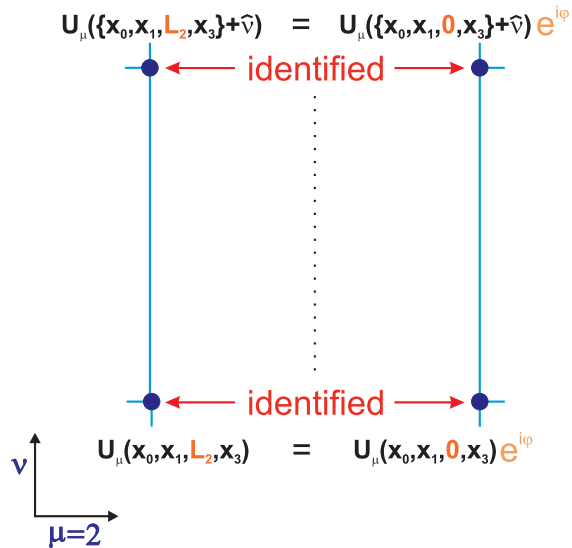
The hypercubic lattice  $L_t \times L_s^3$  ( $L_t < L_s$ ) with the hypertorus geometry was used;  $L_t$  and  $L_s$  are the temporal and the spatial sizes of the lattice, respectively. In the limit of  $L_s \rightarrow \infty$  the temporal size  $L_t$  is related to physical temperature.

The Wilson action of the  $SU(2)$  lattice gauge theory is

$$S_W = \beta \sum_x \sum_{\mu > \nu} \left[ 1 - \frac{1}{2} \text{Tr} U_{\mu\nu}(x) \right]; \quad (6)$$

$$U_{\mu\nu}(x) = U_\mu(x) U_\nu(x + a\hat{\mu}) U_\mu^\dagger(x + a\hat{\nu}) U_\nu^\dagger(x), \quad (7)$$

where  $\beta = 4/g^2$  is lattice coupling,  $g$  is the bare coupling,  $U_\mu(x)$  is the link variable located on the link leaving the lattice site  $x$  in the  $\mu$  direction,  $U_{\mu\nu}(x)$  is the ordered product of the link variables. The effective action  $\bar{S}$  in (5) is the Wilson action  $S_W$  averaged over the Boltzmann configurations produced in the MC simulations.



**Fig. 3:** The plaquette presentation of the twisted boundary conditions

The chromomagnetic flux  $\varphi$  through the whole lattice was introduced by applying the *twisted boundary conditions*. In this approach, the edge links in all directions are identified as usual periodic boundary conditions except for the links in the second spatial direction, for which the additional phase  $\varphi$  is added. The magnetic flux  $\varphi$  is measured in angular units,  $\varphi \in [0; 2\pi)$ .

The MC simulations are carried out by means of the heat bath method. The lattices  $2 \times 8^3$ ,  $2 \times 16^3$  and  $4 \times 8^3$  at  $\beta = 3.0, 5.0$  are considered. These values of the coupling constant correspond to the deconfinement phase and perturbative regime.

The effective action depends smoothly on the flux  $\varphi$  in the region  $\varphi \sim 0$ . So, the free energy density can be fitted by a quadratic function of  $\varphi$ ,

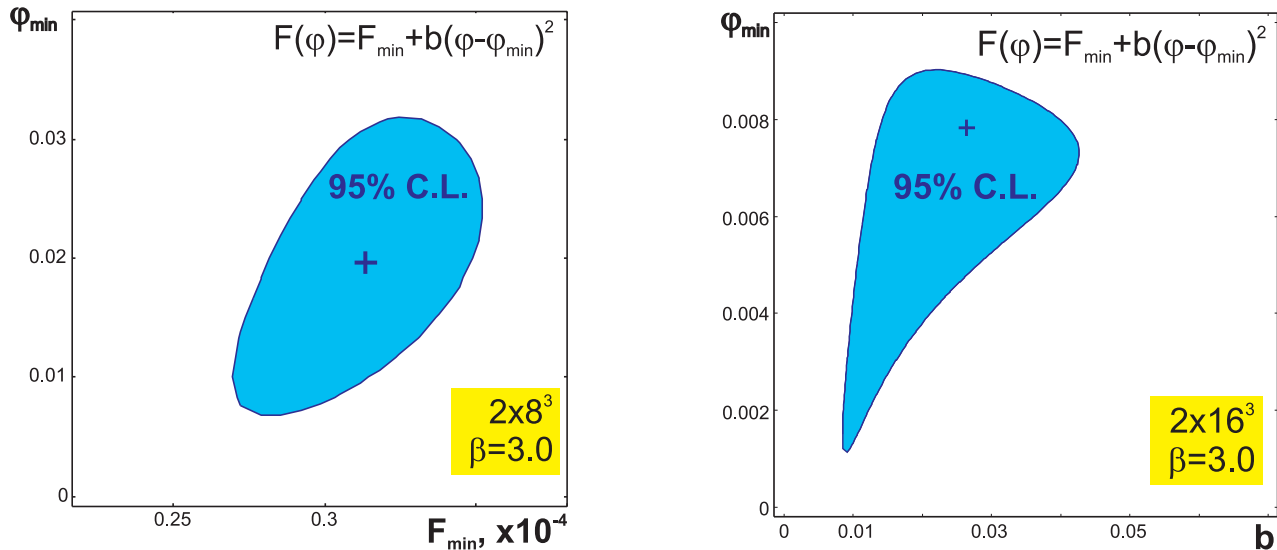
$$F(\varphi) = F_{min} + b(\varphi - \varphi_{min})^2. \quad (8)$$

In Eq.(8), there are three unknown parameters,  $F_{min}$ ,  $b$  and  $\varphi_{min}$ .  $\varphi_{min}$  denotes the minimum position of free energy, whereas the  $F_{min}$  and  $b$  are the free energy density at the minimum and the curvature of the free energy function, correspondingly. They have been fitted by a standard  $\chi^2$  method.

**Table 1: The values of the generated fluxes  $\varphi_{min}$  for different lattices (at the 95% confidence level).**

	$2 \times 8^3$	$2 \times 16^3$	$4 \times 8^3$
$\beta = 3.0$	$0.019^{+0.013}_{-0.012}$	$0.0069^{+0.0022}_{-0.0057}$	$0.005^{+0.005}_{-0.003}$
$\beta = 5.0$	$0.020^{+0.011}_{-0.010}$		

The fit results are given in the Table 1. As one can see,  $\varphi_{min}$  demonstrates the  $2\sigma$ -deviation from zero.



**Fig. 4:** The 95% confidence level area for the parameters  $F_{min}$  and  $\varphi_{min}$  ( $b$  for right fig.).

**The flux  $\varphi_{min}$  is positively determined!**

# MAGNETIC FIELD CHARACTERISTICS

The most essential for what follows characteristics of the field:

## *Stability*

*To verify stability* we substituted the value of  $B_{min}(T)$  in the one-loop EP, the imaginary part was of the order  $10^{-12}$  of the real one.

This means the stable state!

## *Temperature dependence*

In  $SU(2)$  gluodynamics, from the EP

$V(B, T) = V^{(1)}(B, T) + V^{(ring)}(B, T)$  it was determined

$$(gH)^{1/2} = \frac{g^2}{2\pi} T. \quad (9)$$

Thus, a temperature dependent chromomagnetic field is spontaneously created at high temperature.

## *Masslessness (long-range magnetic fields)*

In  $SU(2)$  lattice gauge theory in the presence of Abelian magnetic fields [Antropov, Bordag, Demchik, Skalozub (2010)].

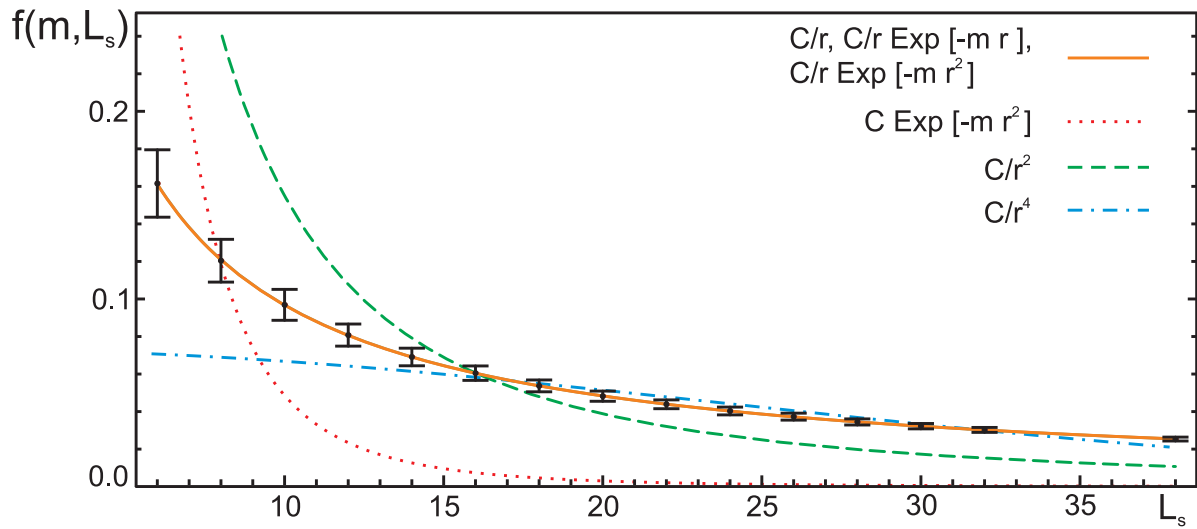
We use the **General Purpose computation on Graphics Processing Units (GPGPU)** technology allowing to study the large lattices up to  $32 \times 64^3$ . Some details of MC simulations on the ATI GPUs can be found in [V. Demchik, A. Strelchenko, arXiv:0903.3053 [hep-lat]].

The constant homogeneous magnetic flux is introduced on a lattice by applying the twisted boundary conditions. For each lattice geometry  $L_t \times L_s^3$ , we have fitted the effect of magnetic field with the lattice plaquette average by means of different functions:

$$\langle U_{untwisted} \rangle - \langle U_{twisted} \rangle = f(m, L_s). \quad (10)$$

**The best fit function** for Abelian magnetic field is  $C/r \exp(-mr)$  **with** a small value of **the magnetic mass**  $m = 0.0000125$ . This case corresponds to the magnetic tube with increasing field strength. Actually, the magnetic mass is equal to zero within the statistical uncertainties appeared.





**Fig. 5:**  $f(m, L_s)$  versus  $L_s$  and fitting curves ( $L_t = 4, \beta = 2.6$ ).

$f(m, L_S)$	$L_S$
$0.161 \pm 0.018$	6
$0.12 \pm 0.011$	8
$0.097 \pm 0.008$	10
$0.081 \pm 0.006$	12
$0.069 \pm 0.005$	14
$0.06 \pm 0.004$	16
$0.054 \pm 0.003$	18
$0.048 \pm 0.003$	20
$0.044 \pm 0.002$	22
$0.04 \pm 0.002$	24
$0.037 \pm 0.002$	26
$0.0345 \pm 0.0016$	28
$0.0322 \pm 0.0014$	30
$0.0302 \pm 0.0013$	32
$0.025 \pm 0.001$	38

**Table 2:**  
**Monte-Carlo data.**

Fit function	Abelian field		
	$\chi^2$	$C$	$m$
$C \exp(-mr)$	901.8	0.063	$m = (2.44^{+0.06}_{-0.06}) \times 10^{-2}$
$C \exp(-m^2 r^2)$	1924.4	0.035	$m = (1.57^{+0.02}_{-0.02}) \times 10^{-2}$
$C/r$	7.090	0.911	
$C/r \exp(-mr)$	7.086	0.912	$m = (1.25^{+52}_{-54}) \times 10^{-6}$
$C/r \exp(-m^2 r^2)$	7.090	0.911	$m^2 = (2.4^{+5951.2}_{-5784}) \times 10^{-10}$
$C/r^2$	31400	28.13	
$C/r^2 \exp(-m^2 r^2)$	7550	18.26	$m^2 = -3.3 \times 10^{-5}$
$C/r^4$	159500	248.9	
$C/r^4 \exp(-m^4 r^4)$	161000	10.0	$m = 0.0$

**Table 3: Fit results for magnetic mass of Abelian magnetic field.**

# QUALITATIVE CONSIDERATION

*The most relevant aspects* of the phenomena of interest are *consequences of asymptotic freedom and spontaneous symmetry breaking* at finite temperature – the basic principles of modern QFT.

*Our main assumption* is that the intergalactic magnetic field had been spontaneously created at high temperature.

This is reasonable because physically the magnetization is the consequence of a large magnetic moment for charged non-Abelian gauge fields (remind the gyromagnetic ratio  $\gamma = 2$  for  $W$ -bosons). Just this property results in the asymptotic freedom of the model.

*First*, in non-Abelian gauge theories magnetic at high temperatures flux conservation does not hold. The vacuum acts as a specific source generating classical fields.

*Second*, the spontaneous vacuum magnetization takes place for small scalar field  $\phi \neq 0$ , only. For the values of  $\phi$  corresponding to any first order phase transition it does not happen.

After the electroweak phase transition, the vacuum polarization ceases to generate magnetic fields and magnetic flux conservation holds. As a result, the familiar dependence on the temperature  $B \sim T^2$  is restored.

*Composite structure of electromagnetic field*  $A_\mu$ . The potentials

$$\begin{aligned} A_\mu &= \frac{1}{\sqrt{g^2 + g'^2}}(g' A_\mu^3 + g b_\mu), \\ Z_\mu &= \frac{1}{\sqrt{g^2 + g'^2}}(g A_\mu^3 - g' b_\mu), \end{aligned} \quad (11)$$

Only the component  $A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} g' A_\mu^3 = \sin \theta_w A_\mu^3$  is present at high temperature. Here  $\theta_w$  is the Weinberg angle,  $\tan \theta_w = \frac{g'}{g}$ .

This is the only component responsible for the intergalactic magnetic field at low temperature.

*In restored phase*,  $b_\mu = 0$ , and  $A_\mu^{(3)}$  is unscreened. This is because the magnetic mass of this field is zero [S. Antropov, M. Bordag, V. Demchik, V. Skalozub arXiv:1011.314/v1 [hep-ph] 13 Nov 2010].

The field is a long range. Its coherence length is to be sufficiently large.

The constituent of the weak isospin field corresponding to the magnetic one is

$$B(T) = \sin \theta_w(T) B^{(3)}(T), \quad (12)$$

where  $B^{(3)}(T)$  is the strength of the field generated spontaneously.

After the phase transition, part of the field is screened.

For EWPT temperature  $T_{ew}$ :

$$\frac{B(T_{ew})}{B_0} = \frac{T_{ew}^2}{T_0^2} = \frac{\sin \theta_w(T_{ew}) B^{(3)}(T_{ew})}{B_0}, \quad (13)$$

$B_0 \sim 10^{-15}G$ . Parameter  $\tau$  can be fixed for given temperature and  $B_0$ . After that, the field strength values at various temperatures can be calculated.

### *Conclusion:*

This is the low bound on the magnetic field strength in the hot universe.

# EFFECTIVE POTENTIAL at HIGH T

The spontaneous vacuum magnetization and zero magnetic mass for the Abelian magnetic fields were determined in lattice simulations [V. Demchik, V. Skalozub (2008)], [S. Antropov, M. Bordag, V. Demchik, V. Skalozub (2010)].

The actual value of  $B(T)$  is close to the one calculated with EP  $V(B, T) = V^{(1)}(B, T) + V^{ring}(B, T)$ . We present analytic results, considering the  $W$ -boson contributions as an example.

*Consider two limits,*

1. weak magnetic field and large scalar field condensate,  $h = eB/M_w^2 < \phi^2$ ,  
 $\phi = \phi_c/\phi_0$ ,  $\beta = 1/T$ ,
2. the case of the restored symmetry,  $\phi = 0$ ,  $gB \neq 0$ ,  $T \neq 0$ .

*For the former*, we show the absence of spontaneous vacuum magnetization at finite temperature.



*For the latter* , we estimate  $B(T)$ . Here  $M_w$  is the  $W$ -boson mass at zero temperature,  $\phi_c$  is a scalar field condensate, and  $\phi_0$  its value at zero temperature.

1. Contribution of  $W$ -bosons,

$$V_w^{(1)}(T, h, \phi) = \frac{h}{\pi^2 \beta^2} \sum_{n=1}^{\infty} \left[ \frac{(\phi^2 - h)^{1/2} \beta}{n} K_1(n\beta(\phi^2 - h)^{1/2}) - \frac{(\phi^2 + h)^{1/2} \beta}{n} K_1(n\beta(\phi^2 + h)^{1/2}) \right] \quad (14)$$

Here  $n$  labels discrete energy values and  $K_1(z)$  is the MacDonald function.

The high temperature limit is the pure Yang-Mills part ( $\tilde{B} \equiv B^{(3)}$ ),

$$V_w^{(1)}(\tilde{B}, T) = \frac{\tilde{B}^2}{2} + \frac{11g^2}{48\pi^2} \tilde{B}^2 \log \frac{T^2}{\tau^2} - \frac{1}{3} \frac{(g\tilde{B})^{3/2} T}{\pi} - i \frac{(g\tilde{B})^{3/2} T}{2\pi} + O(g^2 \tilde{B}^2), \quad (15)$$

where  $\tau$  is a temperature normalization point

## 2. The charged scalar contribution

$$V_{sc}^{(1)}(\tilde{B}, T) = -\frac{1}{96} \frac{g^2}{\pi^2} \tilde{B}^2 \log \frac{T^2}{\tau^2} + \frac{1}{12} \frac{(g\tilde{B})^{3/2} T}{\pi} + O(g^2 \tilde{B}^2), \quad (16)$$

describing the contribution of longitudinal vector components.

The imaginary part is generated because of the unstable mode in the spectrum (2). It is canceled by the term in daisy diagrams for the unstable mode

$$V_{unstable} = \frac{g\tilde{B}T}{2\pi} [\Pi(\tilde{B}, T, n = -1) - g\tilde{B}]^{1/2} + i \frac{(g\tilde{B})^{3/2} T}{2\pi}. \quad (17)$$

Here  $\Pi(\tilde{B}, T, n = -1)$  is the mean value in the ground state  $n = -1$  of the spectrum (2). *If this value is large*, spectrum stabilization takes place.

In the review [V. Demchik, V. Skalozub arXiv:hep-th/9912071 (1999)] the complete EP is present. The mean value of one-loop PT in the spectrum ground state reads,

$$\Pi(\tilde{B}, T, n = -1) = \alpha[12.33 \frac{(g \sin \theta_w B)^{1/2}}{\beta} + i4 \frac{(g \sin \theta_w B)^{1/2}}{\beta}]. \quad (18)$$

Here  $\beta = 1/T$ . Sufficiently large real part stabilizes the spectrum due to radiation corrections included.

# MAGNETIC FIELD at $T_{ew}$

Spontaneous vacuum magnetization at  $T \neq 0$  and non-small  $\phi \neq 0$ .

*Notice*, the magnetization is produced by the gauge field contribution given in Eq. (14). We consider the limit of  $\frac{gB}{T^2} \ll 1$  and  $\phi^2 > h$ . We use the asymptotic expansion of  $K_1(z)$ ,

$$K_1(z) \sim \sqrt{\frac{\pi}{2z}} e^{-z} \left(1 + \frac{3}{8z} - \frac{15}{128z^2} + \dots\right), \quad (19)$$

where  $z = n\beta(\phi^2 \pm h)^{1/2}$ .

Let us investigate the limit of  $\beta \rightarrow \infty$ ,  $\frac{T}{\phi} \ll 1$  and substitute  $(\phi^2 \pm h)^{1/2} = \phi(1 \pm \frac{h}{2\phi^2})$ .

The sum of the tree level energy and (14) reads

$$V = \frac{h^2}{2} - \frac{h^2}{\pi^{3/2}} \frac{T^{1/2}}{\phi^{1/2}} \left(1 - \frac{T}{2\phi}\right) e^{-\frac{\phi}{T}}. \quad (20)$$

The second term is exponentially small and the stationary equation  $\frac{\partial V}{\partial h} = 0$  has the trivial solution  $h = 0$ .

*We conclude:*

**after symmetry breaking the spontaneous vacuum magnetization does not take place.**

At the EWPT temperature the total EP must be used. This can be best done numerically.

To explain the procedure, we consider the part of this EP accounting for the one-loop  $W$ -boson contributions.

The high temperature expansion for the EP coming from charged vector fields is given in (15).

The value of chromomagnetic weak isospin field coming from (15) and (16) is

$$\tilde{B}(T) = \frac{1}{16} \frac{g^3}{\pi^2} \frac{T^2}{\left(1 + \frac{5}{12} \frac{g^2}{\pi^2} \log \frac{T}{\tau}\right)^2}. \quad (21)$$

**We relate this expression with the intergalactic magnetic field  $B_0$ .**

Let us introduce the notations,  $\frac{g^2}{4\pi} = \alpha_s, \alpha = \alpha_s \sin^2 \theta_w, \frac{(g')^2}{4\pi} = \alpha_Y$  and  $\tan^2 \theta_w(T) = \frac{\alpha_Y(T)}{\alpha_s(T)}$ , where  $\alpha$  is the fine structure constant.

For a rough estimate, we substitute:  $\sin^2 \theta_w(T) = \sin^2 \theta_w(0) = 0.23$ .

For the given temperature EWPT,  $T_{ew}$ , the field strength is

$$B(T_{ew}) = B_0 \frac{T_{ew}^2}{T_0^2} = \sin^2 \theta_w(T_{ew}) \tilde{B}(T_{ew}). \quad (22)$$

Assuming  $T_{ew} = 100\text{GeV} = 10^{11}\text{eV}$  and  $T_0 = 2.7\text{K} = 2.3267 \cdot 10^{-4}\text{eV}$ , we obtain

$$B(T_{ew}) \sim 1.85 \cdot 10^{14}G. \quad (23)$$

*This value can serve as a lower bound* on  $B(T)$  at the EWPT. Hence, for the value of  $X = \log \frac{T_{ew}}{\tau}$ , we have the equation

$$B_0 = \frac{1}{2} \frac{\alpha^{3/2}}{\pi^{1/2} \sin^2 \theta_w} \frac{T_0^2}{\left(1 + \frac{5\alpha}{3\pi \sin^2 \theta_w} X\right)^2}, \quad (24)$$

and  $\log \tau$  can be estimated.

To guess the value of  $\tau$  we take  $B_0 \sim 10^{-9}G$ , usually used in cosmology, we obtain  $\tau \sim 300\text{eV}$ .

For the lower bound value  $B \sim 10^{-15}G$  this parameter is much smaller.

*The strong suppression of  $B(T)$  we explain within the SM!*



# MAGNETIC FIELD SCALE

Now we discuss the scale of the field in the restored phase

[E. Elizalde, V. Skalozub, *Europ. Phys. J. C* **72** (2012) 1968.]

*Remind*

If one assumes that after the EWPT the constant field  $B(T_{ew})$  was frozen in the plasma at the Hubble scale,  $R_H(T_{ew})$ , then its comoving coherence scale at present has to be  $\lambda_B(T_0) = 6 \cdot 10^{-4}$  **pc**. This is much smaller than necessary!

**We consider the reheating stage of the universe evolution.** Due to causality, the temperature in the Universe is the same, in all domains of space, which could even be uncorrelated in later moments of time.

*At a given  $T$  the magnetic field generated due to vacuum polarization has the same strength  $B(T)$  everywhere in the Universe.* Formally, the field strength could have different directions, in either external or internal spaces. Different kind of (chromo)magnetic fields can be spontaneously generated. The magnetic fields coherent on huge scales have been present in the early Universe. The origin of this coherence is ensured by the properties of the solution to the field equations discussed above and the causality at the inflationary epoch.

A scenario to produce long-range magnetic field is based on stochastic processes considered already by

*Hogan (1983)*: **The magnetic fields correlated on large scales can be produced by a random walk mechanism, if the magnetic lines generated in some domain of space “forget” about their origin.** The field strength developed on large scales by this process can be estimated as  $B_N \sim B/\sqrt{N}$ , where  $N$  counts the number of domains, with the field  $B$  of a given size, crossed by a magnetic line. The correlation length  $\lambda_B$  in this case can be much larger than the  $R_H(T)$ . It can be estimated as  $\lambda_B(T) \sim NR_H(T)$ .

*In our case* At a given temperature, each uncorrelated domain of space having a Hubble radius  $R_H(T)$  is filled up with a constant magnetic field  $B(T)$ . Its orientation in both external and internal spaces is arbitrary. Hence, a stochastic behavior of the field lines and the appearance of magnetic fields having large correlation lengths  $\lambda_B(T) \geq R_H(T)$  are expected. After the EWPT, these fields evolve according to the flux conservation law  $B(T) \sim T^2$ .

**Note:** *All the fields generated at the inflation epoch are washed out by the vacuum polarization and leave no remnants at present. The reheating stage becomes more important.*

**We also remind:** Long-range nature of the Abelian spontaneously generated magnetic fields is ensured by their zero magnetic mass, what renders these fields unscreened, as is the case for usual  $U(1)$  magnetic fields. The difference, however, is essential because the former fields appear due to vacuum polarization and the latter ones need currents to be produced.

As we have found above, the field strengths at the EWPT temperature, estimated with account to the present-day value of the intergalactic magnetic field strength,  $B_0 \sim 10^{-15} \text{ G}$  or either directly from the vacuum magnetization in the standard model **differ in six orders of magnitude.**

*This huge deviation can be explained by the different scales of the fields considered.*

We used the usual relation between the scale factor and the temperature,

$$\frac{a(T_{ew})}{a(T_0)} = \frac{T_0}{T_{ew}}, \quad (25)$$

taken at the EWPT epoch, and the present-day parameters,  $T_{ew} = 100 \text{ GeV} = 10^{11} \text{ eV}$ ,  $T_0 = 2.3267 \cdot 10^{-4} \text{ eV}$ .

If we assume that  $\lambda_B(T) \sim a(T)$ , then from (25) it follows that  $\lambda_B(T_0) = 6 \cdot 10^{-4}$  **ps**. On the other hand, if one takes  $\lambda_B(T_0) = 1$  **Mpc**, the value  $\lambda_B(T_{ew}) = 2.33 \cdot 10^{-15}$  Mpc is obtained. At the same time, the horizon size is  $a(T_{ew}) = 1.27 \cdot 10^{-24}$  Mpc, thus,  $\lambda_B(T_{ew}) \gg a(T_{ew})$ .

Now, following the idea of Hogan (1983), we relate the size of the correlated field with the random walk process. At  $T_{ew}$ , we have  $\lambda_B(T_{ew}) = Na(T_{ew})$ , hence, we get roughly  $\sqrt{N} = 3 \cdot 10^4$ , and for the field strength “straightened” on the  $N$ -domain scale,  $B_N \sim \frac{B(T_{ew})}{\sqrt{N}}$ . Therefore, accounting for the field strength value calculated for the standard model we obtain  $B_{ls}(T_{ew}) \sim 3 \cdot 10^{15}$  **G** (the subscript in  $B_{ls}$  means “large scale”). This value is close to the value  $B_{ls}(T_{ew}) \sim 2 \cdot 10^{14}$  **G** estimated in Eq. (23).

**In the present scenario, the vacuum polarization generates intergalactic magnetic fields at the reheating without any needs in amplification!**

# CONCLUSION

- At the  $T_{ew}$ , magnetic fields of the order  $B(T_{ew}) \sim 10^{14}G$  did exist.
- The key point is the spontaneous vacuum magnetization, it eliminates the magnetic flux conservation principle at high temperature. Vacuum polarization is responsible for the value of  $B(T)$  at each temperature and serves as a source of it.
- After symmetry breaking,  $\phi$ -condensate suppresses the magnetization.
- Due to stability and zero magnetic mass of the spontaneously created magnetic fields, there are no problem with creating of long-range magnetic fields at high temperature.
- The stochastic random walk process can explain the scale of the present day intergalactic magnetic fields.

These statements change ubiquitous scenario with magnetic flux conservation.